Quiz 2

January 20, 2017

Show all work and circle your final answer.

1. If $\vec{a} = \langle -3, 2, 1 \rangle$ and $\vec{b} = \langle -1, 1, -3 \rangle$, find $\vec{a} \times \vec{b}$.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & 1 \\ -1 & 1 & -3 \end{vmatrix} = \vec{i} \begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} - \vec{j} \begin{vmatrix} -3 & 1 \\ -1 & -3 \end{vmatrix} + \vec{k} \begin{vmatrix} -3 & 2 \\ -1 & 1 \end{vmatrix}$$

$$= \langle -6-1, -(9+1), -3+2 \rangle$$

$$= \boxed{\langle -7, -10, -1 \rangle}$$

2. Find two vectors perpendicular to the plane through the points $(1, 1, 1)$, $(2, -1, 4)$, and $(1, -2, -3)$.

vectors defining the plane: $\langle 2-1, -1-1, 4-1 \rangle$ and $\langle 1-1, -2-1, -3-1 \rangle$

$= \langle 1, -2, 3 \rangle$ and $\langle 0, -3, -4 \rangle$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 3 \\ 0 & -3 & -4 \end{vmatrix} = \vec{i} \begin{vmatrix} -2 & 3 \\ -3 & -4 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 3 \\ 0 & -4 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & -2 \\ 0 & -3 \end{vmatrix} = \boxed{\langle 17, +4, -3 \rangle}$$

and $\boxed{\langle -17, -4, 3 \rangle}$

3. Find the area of the triangle in the xy-plane with vertices $(1, -1)$, $(0, -2)$, and $(3, 0)$.

vectors (in $\mathbb{R}^3$) defining the triangle: $\langle 1-0, -1-(-2), 0 \rangle$ and $\langle 3-0, 0-(-2), 0 \rangle$

$= \langle 1, 1, 0 \rangle$ and $\langle 3, 2, 0 \rangle$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \vec{i} \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= \langle 0, 0, -1 \rangle$$

So $A_{\square} = |\langle 0, 0, -1 \rangle| = 1$ and $A_{\triangle} = \frac{1}{2} A_{\square} = \boxed{\frac{1}{2}}$

4. If $|\vec{a}| = 1$, $|\vec{b}| = 5$, and $\vec{a} \times \vec{b} = \langle 1, -2, 2 \rangle$, are $\vec{a}$ and $\vec{b}$ parallel, orthogonal, or neither?

$$\sin\theta = \frac{|\langle \vec{a} \times \vec{b} \rangle|}{|\vec{a}||\vec{b}|} = \frac{\sqrt{1^2+2^2+2^2}}{1 \cdot 5} = \frac{3}{5}$$

$\theta = \sin^{-1}\left(\frac{3}{5}\right)$, so $\theta \neq 0$ or π

Neither